On Infinite Words Determined by L Systems

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WORDS 2013

Outline

- Background: L systems and infinite words.
- Framework: Prefix languages which determine infinite words.
- Techniques: Infinite subsets of L systems.
- Results: Classes of infinite words determined by L systems.

L Systems

- parallel rewriting systems
- introduced by Aristid Lindenmayer in 1968
- original purpose was to model the development of simple organisms
- applications in computer graphics, fractals, artificial life
- references: [Rozenberg & Salomaa 1980]; [Kari, Rozenberg, & Salomaa 1997]

D0L Systems

- D0L = “deterministic Lindenmayer system with 0 symbols of context”
- A D0L system is a tuple G = (A,h,w) where
  - A is an alphabet,
  - h is a morphism on A, and
  - w (the “axiom” or start string) is in A*.
- The language of G is $L(G) = \{h^i(w) \mid i \geq 0\}$.

D0L Words

- If $h(w) = wx$ and x is not mortal, we say that h is "prolongable" on w.
- $w \to w\,x \to w\,x\,h(x) \to w\,x\,h(x)\,h^2(x) \to \ldots$
- Let $\alpha = h^\omega(w) = w\,x\,h(x)\,h^2(x)\,h^3(x) \ldots$
- We call α an infinite D0L word.

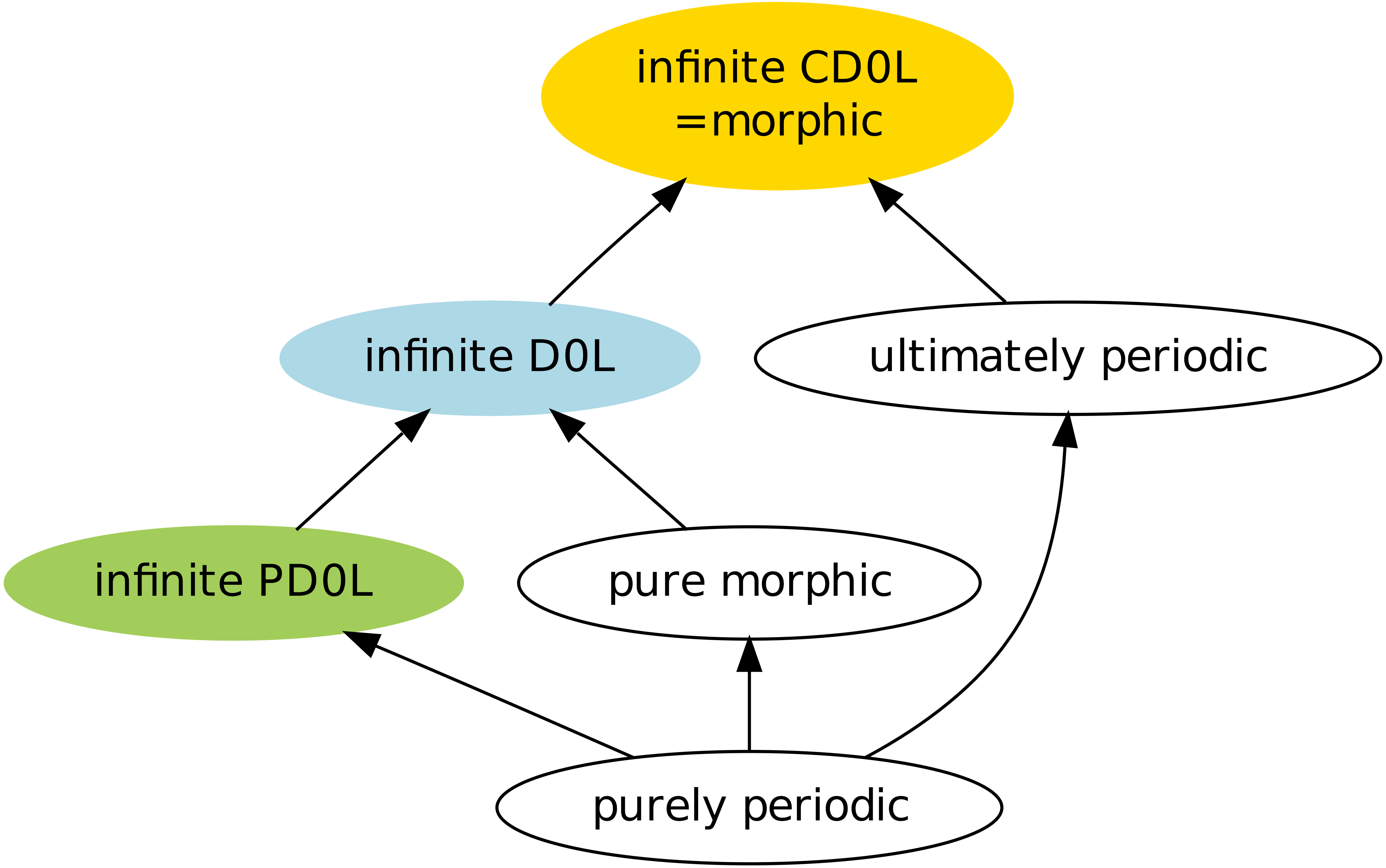
D0L Example

- Let $G = (A,h,w)$, where
  - $A = \{0,1\}$,
  - $h(0) = 01$ and $h(1) = 10$, and
  - $w = 0$.
- $0 \rightarrow 01 \rightarrow 0110 \rightarrow 01101001 \rightarrow \ldots$
- $\alpha = 01101001\ldots$ (Thue-Morse word)

PD0L and CD0L

- A PD0L system is a D0L system for which the morphism h is nonerasing ("propagating").
  - If h is prolongable on w, we call $h^\omega(w)$ an infinite PD0L word.
- A CD0L system is a tuple G = (A,h,w,e) where G′ = (A,h,w) is a D0L system and e is a coding on A. The language of G is L(G) = {e(s) | s is in L(G′)}.
  - If h is prolongable on w, we call $e(h^\omega(w))$ an infinite CD0L word.

Infinite Words



Some Previous Results

- “Descriptional complexity” of infinite words
  - [Culik & Salomaa 1982] On infinite words obtained by iterating morphisms
  - [Pansiot 1985] On various classes of infinite words obtained by iterated mappings
  - [Culik & Karhumäki 1994] Iterative devices generating infinite words

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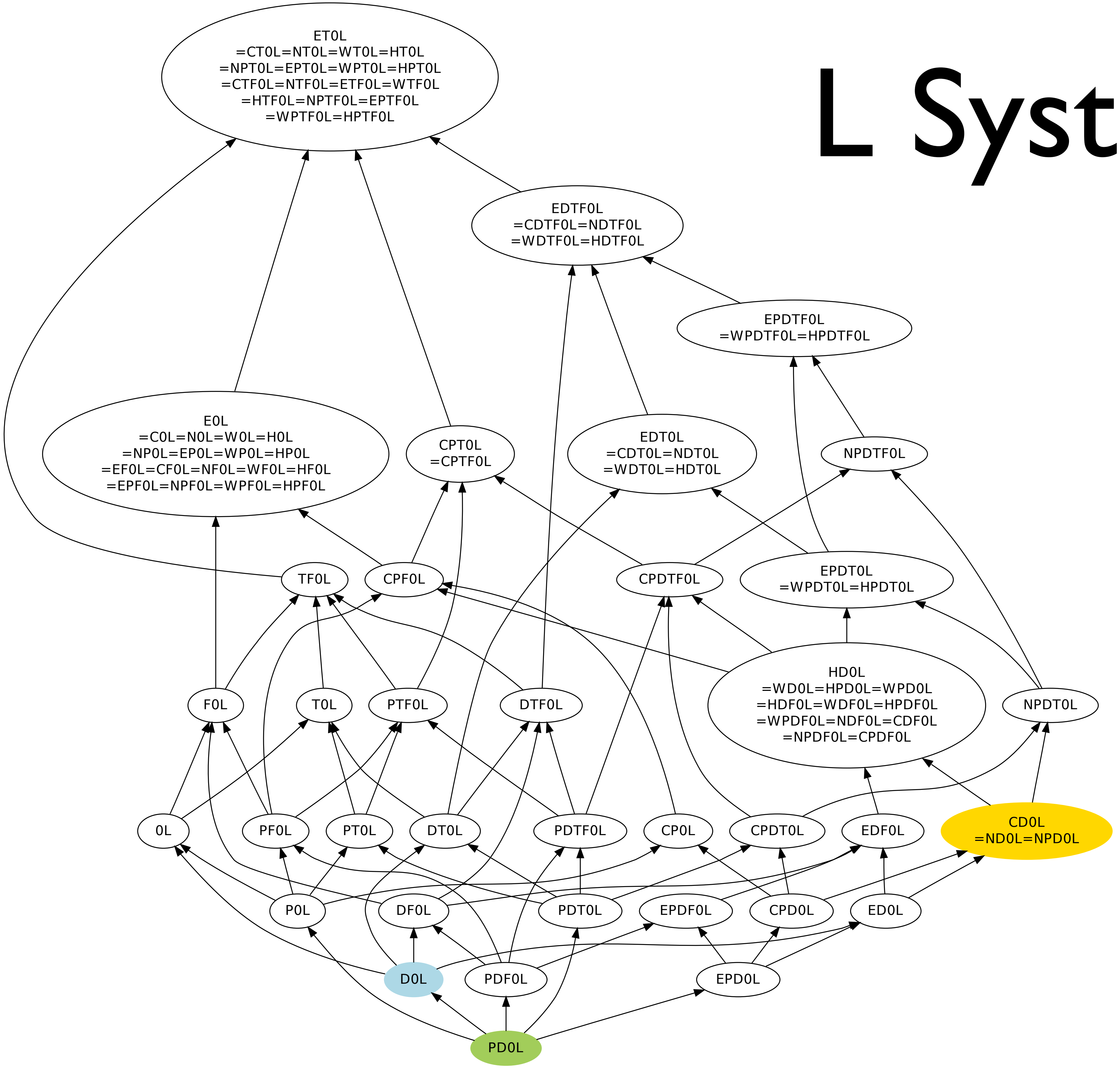
L Systems



ET0L
=CT0L=NT0L=WT0L=HT0L
=NPT0L=EPT0L=WPT0L=HPT0L
=CTF0L=NTF0L=ETF0L=WTF0L
=HTF0L=NPTF0L=EPTF0L
=WPTF0L=HPTF0L
EDTF0L
=CDTF0L=NDTF0L
=WDTF0L=HDTF0L
EPDTF0L
=WPDTF0L=HPDTF0L
E0L
=C0L=N0L=W0L=H0L
=NP0L=EP0L=WP0L=HP0L
=EF0L=CF0L=NF0L=WF0L=HF0L
=EPF0L=NPF0L=WPF0L=HPF0L
CPT0L
=CPTF0L
EDT0L
=CDT0L=NDT0L
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CD0L
=ND0L=NPD0L
P0L
DF0L
PDT0L
EPDF0L
CPD0L
ED0L
D0L
PDF0L
EPD0L
PD0L

Nondeterminism

- A 0L system introduces nondeterminism by replacing the morphism h with a finite substitution σ.
- A DT0L system replaces the morphism h with a set H of morphisms or "tables".
- In each case, there is no longer just one possible derivation sequence.
- Infinite 0L and DT0L words?

Prefix Languages

- [Book 1977] On languages with a certain prefix property
- We say that a language L determines an infinite word α if L is infinite and every x in L is a prefix of α.
- L is called an infinite prefix language.
- Example: $L = \{ab, abab, ababab, \ldots\}$ determines $\alpha = (ab)^\omega = ababab\ldots$

$\omega(C)$

- Where C is a class of languages, let $\omega(C) = \{\alpha \mid \text{some } L \text{ in } C \text{ determines } \alpha\}$.
- $\omega(C)$ is the set of infinite words determined by the languages in C.
- We can refer to $\omega(0L)$ as "infinite 0L words", $\omega(DT0L)$ as "infinite DT0L words", etc.

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Infinite Subsets

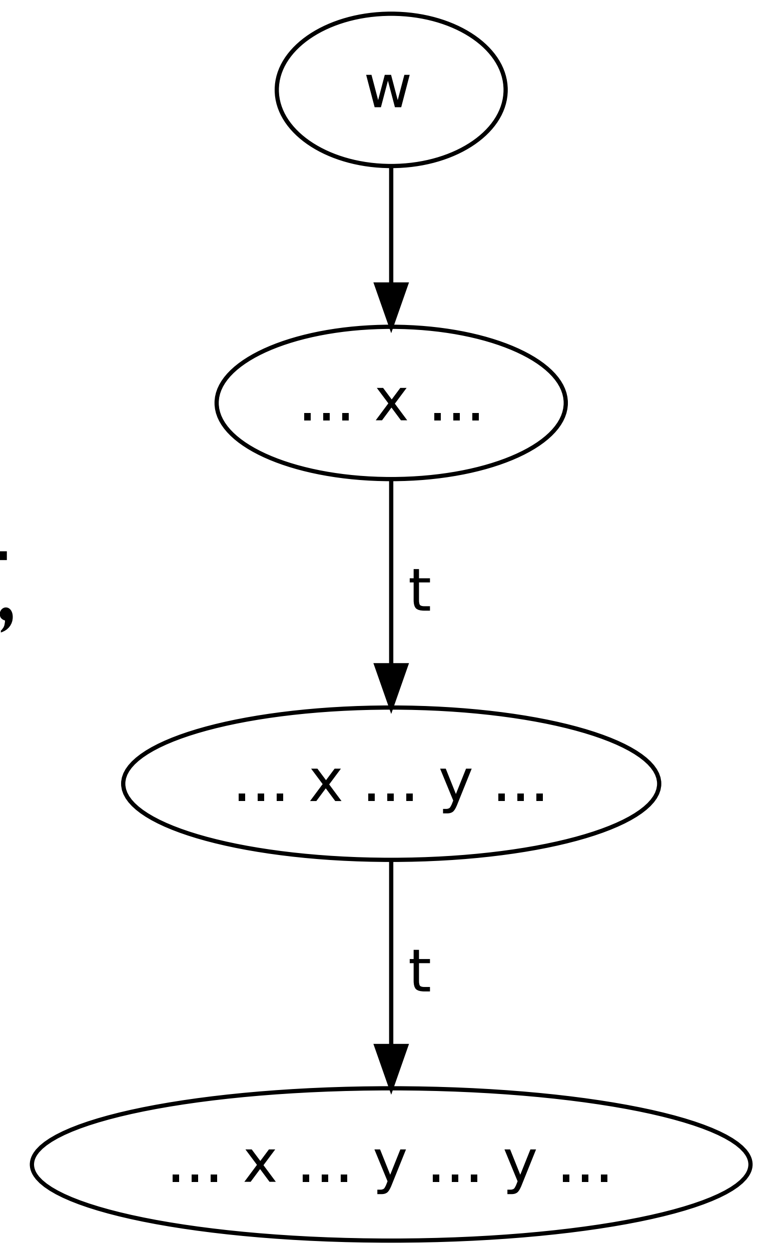
- If a language L determines an infinite word α and L' is an infinite subset of L, then L' determines α.
- Take two language classes C and C' such that $C' \subseteq C$.
- If for every infinite language L in C, there is an infinite language L' in C' such that $L' \subseteq L$, then $\omega(C') = \omega(C)$.
- By categorizing L systems with regard to their infinite subsets, we can categorize infinite words determined by L systems.

T0L Systems

- A T0L system is a tuple G = (A,T,w) where A is an alphabet, T is a finite nonempty set of finite substitutions on A (called "tables"), and w is in A^*.
- The language of G is L(G) = {s | s is in $\sigma_i \ldots \sigma_1(w)$ for some $i \geq 0$ and $\sigma_1,\ldots,\sigma_i$ in T}.
- T0L pumping lemma due to [Smith 2013] Infiniteness and boundedness in 0L, DT0L, and T0L systems

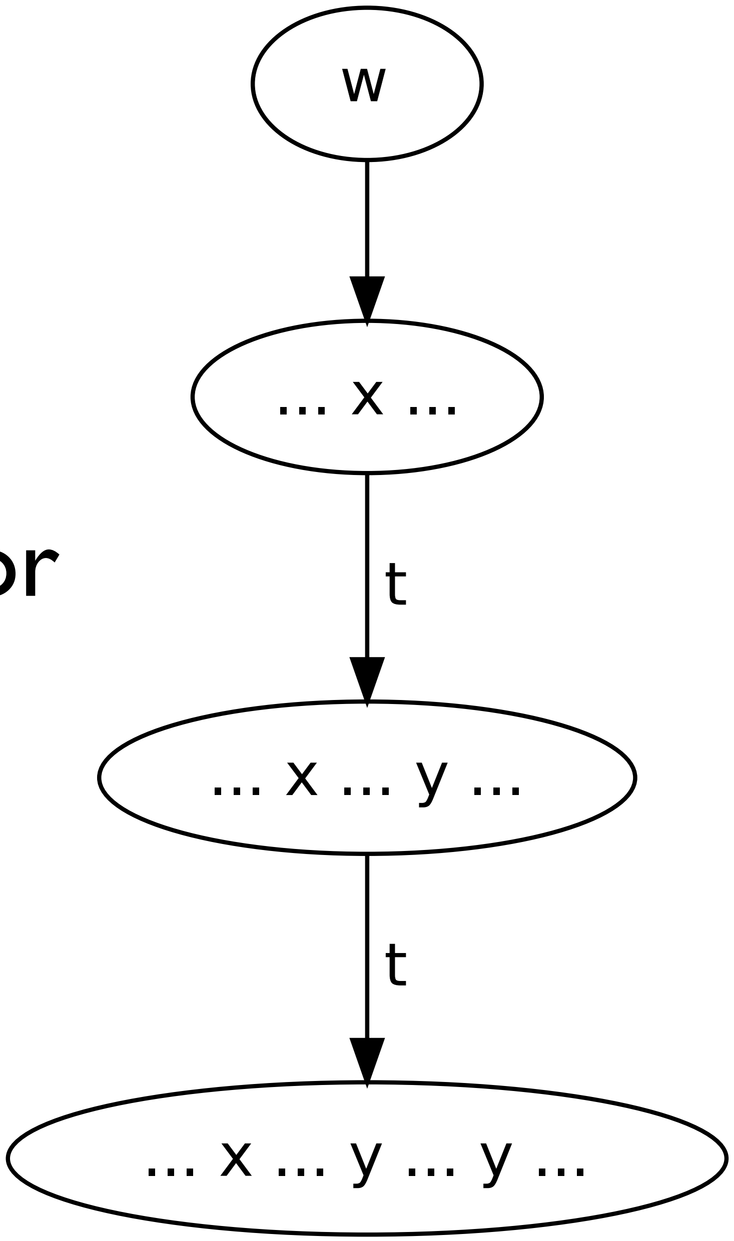
T0L Pumping Lemma

- For every infinite T0L system G = (A,T,w), there are are x,y in A such that
  - some s_0 in L(G) contains x, and
  - for some composition t of tables from T,
    - t(x) includes a string s_1 containing distinct occurrences of x and y and
    - t(y) includes a string s_2 containing y.



Infinite D0L Subset

- Corollary of T0L pumping lemma: Every infinite T0L language has an infinite D0L subset.
- Proof idea: Let $h(x) = s_1$, $h(y) = s_2$, and for every other c in A, h(c) = any s in t(c).
- Then the language of the D0L system (A,h, s_0) is an infinite subset of L(G).



More Subsets

- Other key subset relationships we obtain, and use to categorize the hierarchy of L systems:
  - Every infinite PT0L language has an infinite PD0L subset.
  - Every infinite ET0L language has an infinite CD0L subset.
  - Every infinite ED0L language has an infinite D0L subset.

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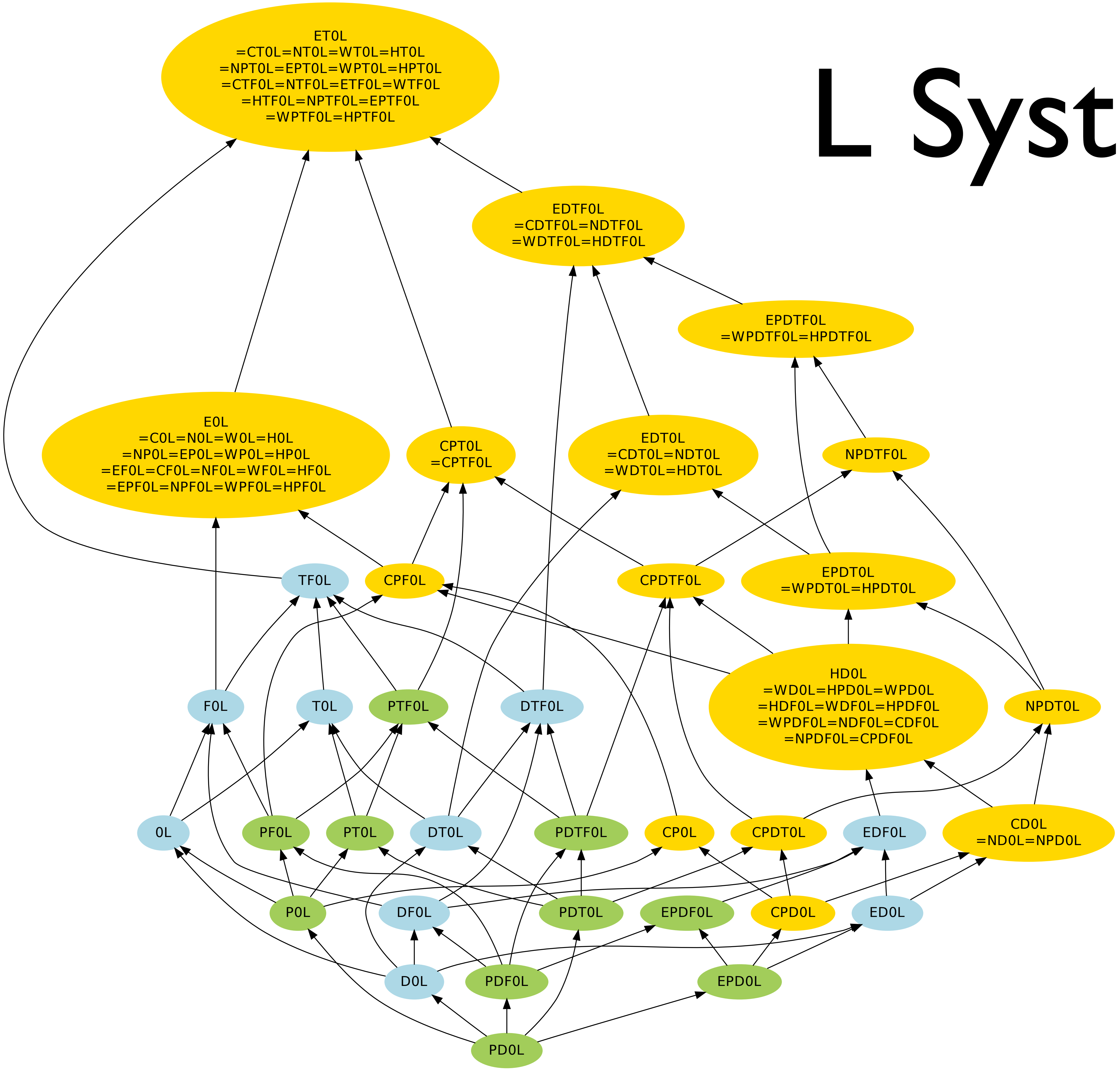
L Systems


ET0L
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EPDTF0L
=WPDTF0L=HPDTF0L
E0L
=C0L=N0L=W0L=H0L
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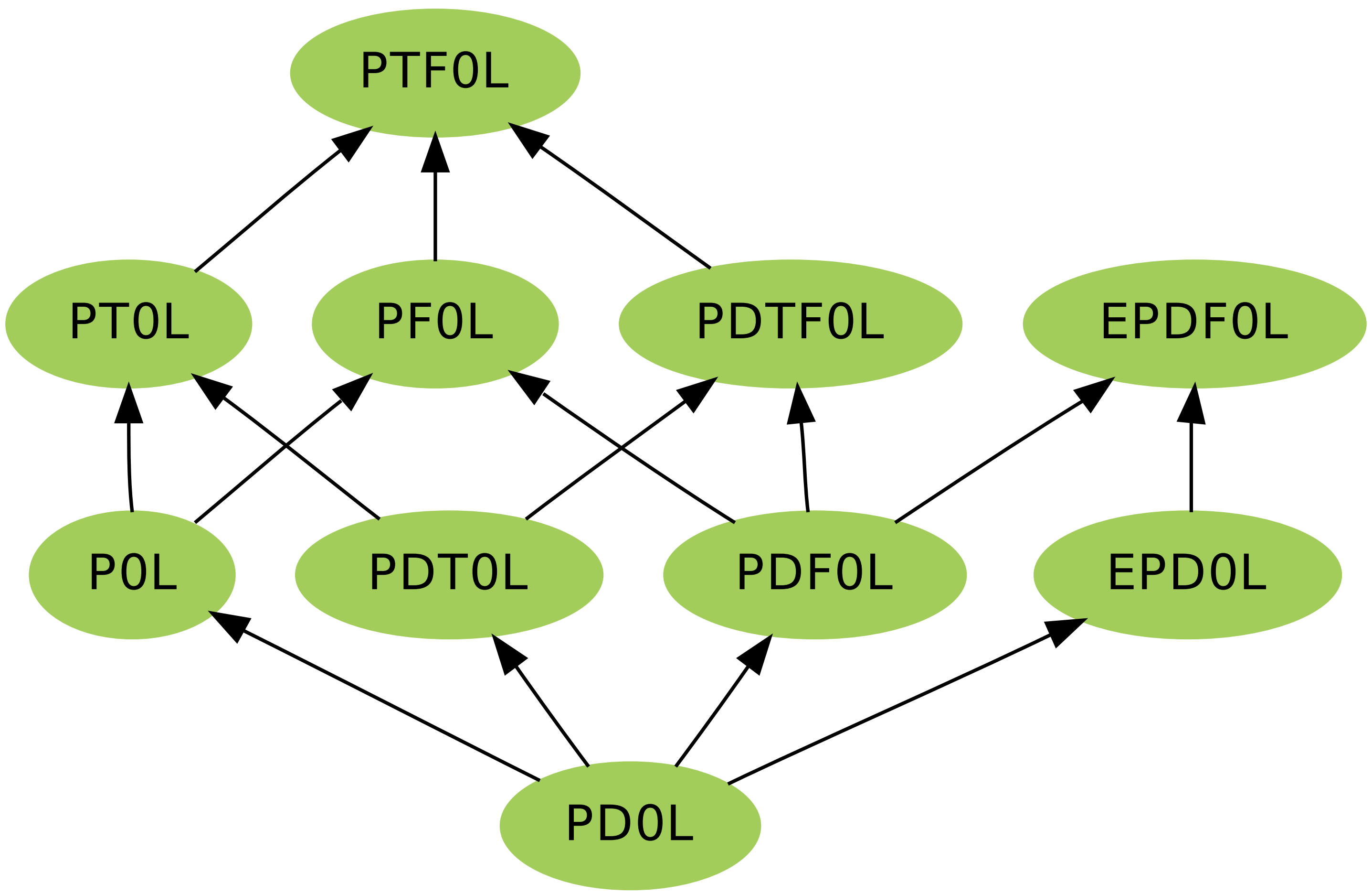
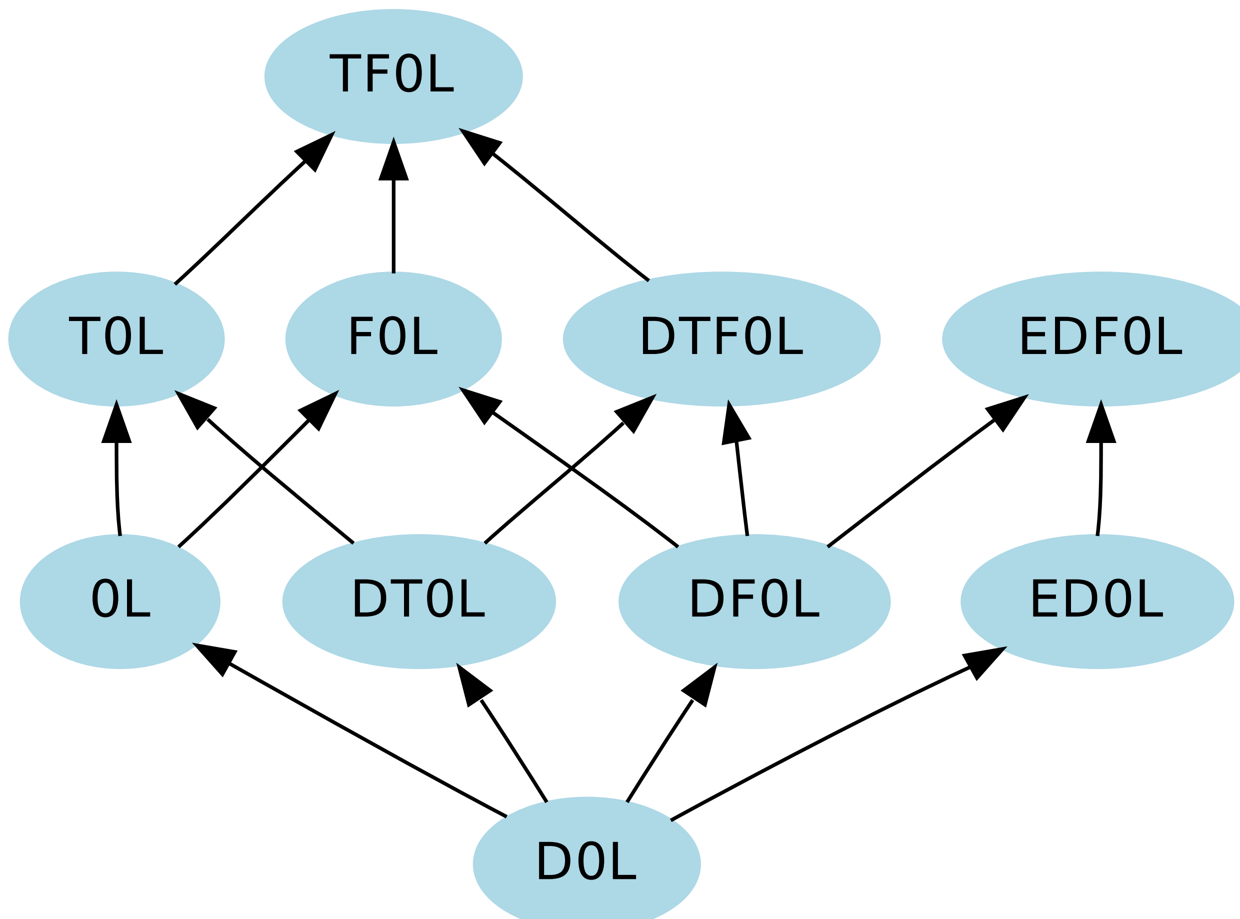
PD0L

ω(PD0L)



For each class C, every infinite C language has an infinite PD0L subset, and ω(C) = ω(PD0L).

ω(D0L)



For each class C, every infinite C language has an infinite D0L subset, and ω(C) = ω(D0L).

ω(CD0L)

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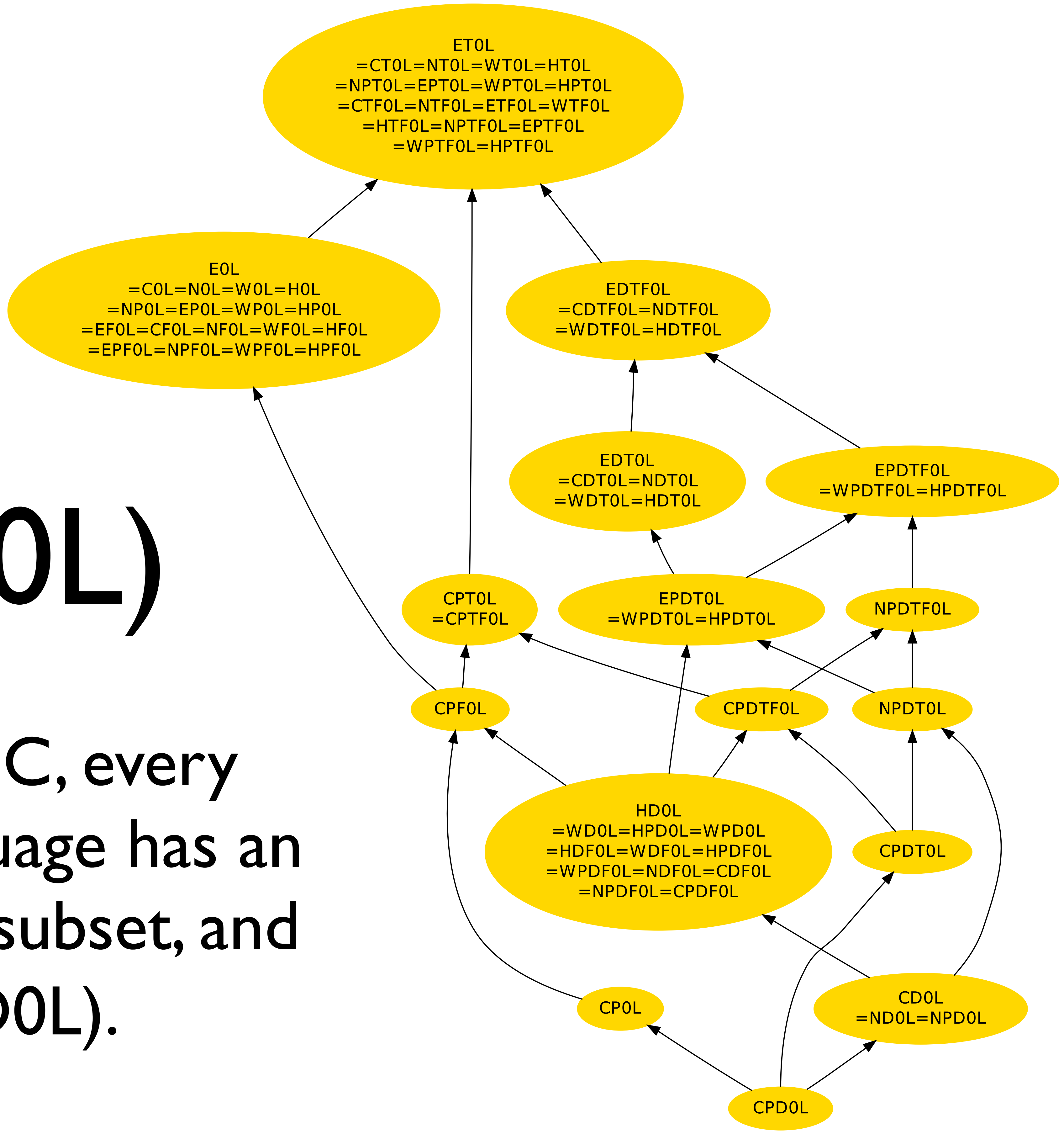

Future Work

- Investigate other language devices (e.g. automata, grammars) to see what infinite words their prefix languages determine.
- Investigate other infinite words to see in what prefix language classes they can be determined.

Thank you!